Substitutability between Home and School in Child Human Capital Production: Aggregate and Distributional Consequences of Education Policy

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Abstract

The elasticity of substitution σ^g between public and private inputs drives many policy results in quantitative life-cycle models of parental investment. Empirical evidence does not discipline it. In a model calibrated to US data, σ^g governs a trade-off between aggregate and distributional effects of education policy, and the nesting order of the production function affects their size. Moving from unit-elastic to perfectly substitutable inputs nearly halves the average welfare loss from a one-year cut in public schooling. The same parameter shift raises the loss of children of high-school dropouts from about 1.5 to 2 times that of children of college graduates.

Keywords: human capital; public education; parental investment; substitution elasticity; school closures
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1 Introduction

Public investment in education is one of the largest and most debated levers of social policy (OECD, 2026), pursued with two main objectives: raising average human capital and reducing inequality of opportunity (Glomm and Ravikumar, 1992; Bénabou, 2002). Whether it achieves either depends critically on how families respond, specifically on whether they view public schooling as a substitute for, or a complement to, their own investments in children. The parameter governing this response, the elasticity of substitution σ^g between public and private inputs in the human capital production function, is therefore central to any quantitative evaluation of education policy.

Two largely separate research programs inform our understanding of this production function. The Child Development (CD) literature (Cunha et al., 2006; Cunha and Heckman, 2007; Cunha et al., 2010) models skill formation as a dynamic process driven by parental time and monetary investments, emphasizing self-productivity and dynamic complementarity, but largely abstracting from public schooling as a direct input. The Education Production Function (EPF) literature uses experimental and administrative data to estimate the value added of classrooms and teachers (Krueger, 1999; Chetty et al., 2014), but pays limited attention to how school inputs interact with parental investments. A more recent strand bridges the two by incorporating both into a unified production function (Kotera and Seshadri, 2017; Lee and Seshadri, 2019; Fuchs-Schündeln et al., 2022, 2023), making σ^g a key structural parameter.

Despite its importance, σ^g is not disciplined by data: to date, there exist no credible micro estimates of this parameter in the context of child skill formation. Caucutt et al. (2026) focus on child care. They estimate substitution elasticities between private inputs (parental time, goods, and market-based child care), find moderately strong complementarity, and explicitly note that no study estimates the substitutability between family-based and school-based inputs. Their identification strategy requires variation in relative input quantities. Market child care has such variation; public schooling is largely fixed in quantity by compulsory attendance. Variation in school quality exists: Agostinelli et al. (2025) exploit latent classroom fixed effects to estimate a home-school production function for US children during kindergarten. In their most general specification, the cross-elasticity between home and classroom inputs is imprecisely estimated. Cross-district variation in public school spending offers another route (Kotera and Seshadri, 2017), though data on private

parental investment are not available at the district level, making this feasible only within a structural model.[1] Beyond σ^g, the nesting order of the production function (which inputs substitute against each other at which level) is equally undisciplined. In Fuchs-Schündeln et al. (2022), i^g substitutes against the full parental input composite; moving it down by one nest, so that it substitutes against parental time alone, halves the upper bound of the elasticity of acquired human capital with respect to public investment, from $\frac{1}{2}$ to $\frac{1}{4}$ at calibrated parameter values.

As a laboratory, I use a quantitative life-cycle model of the class just described. Parents invest time and money alongside a public schooling input, and the model is calibrated to US household microdata and aggregate data (Fuchs-Schündeln et al., 2022). I vary σ^g across three cases: the baseline ($\sigma^g = 2.43$, from Kotera and Seshadri 2017), perfect substitutes ($\sigma^g = \infty$), and Cobb-Douglas ($\sigma^g = 1$). The experiment is a one-time reduction in the public schooling input, of the magnitude of the 2020 Covid-19 closures, and I trace its long-run effects on the human capital, educational attainment, and welfare of children. The closures are a policy-driven cut in the public input, and the dependence on σ^g that I document applies to any change in it: temporary or permanent, a funding cut or an expansion. The main finding is a trade-off governed by σ^g: a higher value reduces the average welfare loss, as parents substitute toward their own investment, but steepens its socioeconomic gradient in proportional terms, as better-educated, higher-income parents substitute more. Aggregate efficiency and distributional equity move in opposite directions in a parameter that existing evidence does not pin down.

2 Model

I use the model of Fuchs-Schündeln et al. (2022) and Fuchs-Schündeln et al. (2023), which give the full exposition.

The core of the model is a multi-stage human capital production function for children. At each stage j of childhood, human capital evolves as

$$h'(j) = \left(\kappa_j^h \, h(j)^{1-\frac{1}{\sigma_j^h}} + (1-\kappa_j^h) \, I(j)^{1-\frac{1}{\sigma_j^h}} \right)^{\frac{1}{1-\frac{1}{\sigma_j^h}}}, \tag{1}$$

[1] These approaches also do not translate directly into σ^g for the life-cycle models considered here, which treat public schooling as a single composite input i^g with no quantity–quality distinction.

where $h(j)$ is human capital at the beginning of stage j, $I(j)$ is a composite investment input, κ_j^h governs the relative weight of existing human capital (self-productivity), and σ_j^h is the elasticity of substitution between existing stock and new investment. The composite investment $I(j)$ aggregates public schooling i^g and a private parental input i^p as

$$I(j) = \bar{A}_i \left(\kappa^g \left(i^g\right)^{1-\frac{1}{\sigma^g}} + (1-\kappa^g) \left(i^p\right)^{1-\frac{1}{\sigma^g}} \right)^{\frac{1}{1-\frac{1}{\sigma^g}}}, \tag{2}$$

where σ^g is the elasticity of substitution between public and private inputs and κ^g is the relative weight on the public input. When $\sigma^g > 1$ the two inputs are substitutes; when $\sigma^g < 1$ they are complements; $\sigma^g = 1$ corresponds to Cobb-Douglas aggregation. Equation (2) is the upper investment nest of Fuchs-Schündeln et al. (2022) (with age-dependent weights and input normalizations suppressed); the parental composite i^p is itself a CES aggregate of monetary investment i^m and parental time i^t, with relative weight κ^m and unit elasticity. The level at which i^g enters the nesting is itself an undisciplined modeling choice, as the following remark shows.

Remark 1. Nesting order and the elasticity of human capital w.r.t. public investment. *The elasticity of acquired human capital $\epsilon_{h',i^g} \equiv \frac{\partial h'}{\partial i^g}\frac{i^g}{h'}$, evaluated at equal input ratios with $\bar{A}_i = 1$, equals $(1-\kappa^h)\,\kappa^g$ under this nesting; with weights near one-half it lies in $(0, \frac{1}{2})$. Under an alternative nesting that places i^g one level lower, so that it substitutes against parental time i^t alone, the same elasticity becomes a product of three weight complements, one for each nest separating i^g from h', and lies in $(0, \frac{1}{4})$: the upper bound halves. The nesting order is not disciplined by data, yet it determines the magnitude of the effect of public education spending on average human capital and wages. Together with σ^g, it constitutes a second dimension of functional-form uncertainty with first-order quantitative consequences.*

Parents are heterogeneous in educational attainment $s \in \{\text{no}, \text{hs}, \text{co}\}$, assets a, and marital status m. For expositional clarity, I present the problem for a single parent and drop marital status m throughout; the actual model computations are based on the full population of singles and couples, as in Fuchs-Schündeln et al. (2022). During the years in which children are present in the household, parents choose consumption c, monetary investment i^m, time investment i^t, next-period assets a', and next-period

child human capital h' to solve

$$V(j,s,\eta,\varepsilon;a,h) = \max_{c,\,i^m,\,i^t,\,a',\,h'} \left\{ u\left(\frac{c}{1+\zeta_c\,\xi(s)}\right) - v\big(\ell + \kappa\,\xi(s)\,i^t\big) \right. \\ \left. + \beta\,\mathbf{E}_{\eta',\varepsilon'}\,V(j{+}1,s,\eta',\varepsilon';a',h')\right\} \quad (3)$$

subject to

$$\begin{aligned} a' + c(1+\tau^c) + \xi(s)\,i^m &= a(1+r(1-\tau^k)) + y(1-\tau^p) - T(y(1-0.5\tau^p)) \\ y &= w\,\epsilon(s,j)\,\eta\,\varepsilon\,\ell \\ a' &\geq -\underline{a}(j,s) \\ h' &= g(j,h,I) \\ I &= I(i^m,i^t,i^g), \end{aligned}$$

where $\xi(s)$ is the number of children per household, κ is the relative disutility weight on time investments, and the composite input $I(i^m,i^t,i^g)$ is delivered by equation (2) together with the parental sub-nest i^p, feeding the human capital technology $g(\cdot)$ of equation (1). When children leave the household, parents make a one-time inter-vivos transfer b to their children, which enters the education and lifetime consumption-saving problem of the children. The government provides a fixed per-child public schooling input i^g.

The key parameter for the results that follow is σ^g, which governs how easily parents can substitute their own investments for the public schooling input i^g. In the baseline calibration, $\sigma^g = 2.43$, taken from Kotera and Seshadri (2017). I compare this to two further cases: perfect substitutability ($\sigma^g = \infty$) and Cobb-Douglas ($\sigma^g = 1$). The thought experiment is a one-time, unexpected one-year reduction in i^g, which represents the Covid-19 school closures, and I trace its long-run consequences for the human capital, education, earnings, and welfare of children. As in Fuchs-Schündeln et al. (2022), the experiment is conducted in partial equilibrium: factor prices and fiscal policy are held at their pre-shock values, so the public resources freed by the one-year closure are absorbed by government debt borne by future generations and are not rebated to the affected households.

3 Results

The baseline value $\sigma^g = 2.43$ is the one used in both Fuchs-Schündeln et al. (2022) and Fuchs-Schündeln et al. (2023). The perfect-substitutes and Cobb-Douglas cases bracket it from above and below.

3.1 Average Effects

Parental Human Capital Investment Responses. With perfect substitutability ($\sigma^g = \infty$), parents find it easier to compensate for the lost public input, so they raise both monetary and time investment far more than in the baseline (Table 1). Average human capital at age 16 then falls by 1.9%, compared with 3.3% in the baseline, the share of college graduates by 1.2 compared with 2.3 percentage points, and the present discounted value of gross lifetime earnings by 1.2% compared with 2.1%. Inter-vivos transfers move in the opposite direction, as parents shift resources toward in-childhood investment. Under Cobb-Douglas aggregation ($\sigma^g = 1$), which is unit-elastic and less substitutable than the baseline, the responses reverse: the loss of public schooling now lowers the marginal return to private investment, so both investment margins fall, transfers rise, and average human capital drops by 4.7% (the college share by 3.5 percentage points and lifetime earnings by 3.0%).

Welfare. The average welfare loss, measured as the consumption-equivalent variation (CEV), ranges from -0.81% under perfect substitutability to -1.56% under Cobb-Douglas, with the baseline at -1.21%. The near-doubling of the welfare loss as one moves from perfect substitutes to Cobb-Douglas illustrates how consequential the choice of σ^g is for any quantitative assessment of public education policy.

3.2 Distributional Effects

While a higher σ^g lowers the average welfare loss, it widens the gap in losses between children of less- and more-educated parents. Table 2 shows the CEV welfare loss of the children affected by the closures, broken down by parental education. In the baseline, children of high-school dropout parents suffer welfare losses roughly 65% larger than those of college-educated parents (-1.47% vs. -0.89%). Under perfect substitutability this gap widens to 102%: richer parents respond more strongly to the

Table 1: Varying Elasticity of Substitution between Public and Parental Inputs

Variable / Elasticity	$\sigma^g = 2.43$	$\sigma^g = \infty$	$\sigma^g = 1$
	change in p.p.		
share no high school	1.92	1.38	2.40
share high school	0.41	-0.21	1.08
share college	-2.34	-1.17	-3.48
	change in %		
Avg. human capital (age 16)	-3.32	-1.89	-4.65
PDV of gross earnings	-2.10	-1.17	-2.97
Child CEV	-1.21	-0.81	-1.56
	change in %		
Avg. monetary investment	14.06	29.39	-1.81
Avg. time investment	6.73	15.54	-0.85
Avg. inter-vivos transfer	0.73	-0.49	2.01

Notes: All changes are relative to the pre-shock steady state. The first column is the baseline calibration. PDV: present discounted value of gross lifetime earnings. CEV: consumption-equivalent variation. Inter-vivos transfers are made when children leave the household.

school closure shock because they have greater financial resources and, due to dynamic complementarity in the human capital production function, face higher marginal returns to investing in their already higher-ability children. Higher substitutability thus amplifies the pre-existing socioeconomic gradient. The reverse holds under Cobb-Douglas aggregation: the gap narrows to 54% (-1.80% vs. -1.17%), as the lower substitutability between public and private inputs limits the ability of all parents, rich and poor alike, to offset the shock with their own investment. Throughout, the gradient is measured in proportional terms. This is the welfare-relevant comparison, since the CEV already expresses the loss of each group relative to its own lifetime consumption.

3.3 Trade-Off

Together the two tables reveal a trade-off. A higher σ^g reduces the average welfare loss from the cut in public schooling, as parents undo more of the damage, but steepens the socioeconomic gradient in that loss. As σ^g rises, aggregate efficiency improves and distributional equity deteriorates, and both changes are large.

Table 2: Varying Elasticity of Substitution: Welfare Loss of Children by Parental Education

Parental Education / Elasticity	$\sigma^g = 2.43$	$\sigma^g = \infty$	$\sigma^g = 1$
	CEV change in %		
Parental Education $s =$ no	-1.47	-1.11	-1.80
Parental Education $s =$ hs	-1.27	-0.80	-1.66
Parental Education $s =$ co	-0.89	-0.55	-1.17

Notes: Columns as in Table 1. Each entry reports the welfare loss, measured as consumption-equivalent variation (CEV), of children affected by the school closures (aged 4–14 at the time of the shock), by parental education group (no high school completion, high school completion, college completion). The CEV is the uniform percentage change in lifetime consumption that makes the average affected child in each group indifferent between the outcomes with and without the shock, evaluated at labor market entry.

4 Conclusion

The substitutability between home and school inputs governs a trade-off between the aggregate and distributional consequences of education policy, and the policy implication is uncomfortable: whether a high σ^g is "good news" depends entirely on the objective. From an efficiency standpoint high substitutability is desirable: parents undo much of an adverse public-investment shock, limiting aggregate welfare losses. From an equity standpoint it is undesirable, because richer, better-educated parents respond more strongly and the burden of adjustment falls on disadvantaged families. A high σ^g makes public education spending look less necessary in the aggregate. Distributionally, the same σ^g makes public spending most important. When the two inputs are less substitutable, public spending does double duty: it raises average human capital and compresses the socioeconomic gradient in proportional terms. Yet σ^g is not pinned down: no available design delivers a model-free estimate of the substitutability between parental and school-based inputs, and the nesting order compounds the uncertainty. Evaluations that fix σ^g by assumption (perfect substitutes, Cobb-Douglas, or a borrowed structural estimate) therefore risk misjudging both the magnitude and the direction of the response to public education spending.

Data Statement

This paper does not use original data. The model is calibrated to US household microdata and aggregate data as described in Fuchs-Schündeln et al. (2022); all data moments and calibrated parameters are reported in that paper and its online appendix. Replication code and model output are available from the author upon request.

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References

Agostinelli, F., M. Saharkhiz, and M. Wiswall (2025). Home and School in the Development of Children. *International Economic Review 66*(2), 527–566.

Bénabou, R. (2002). Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency? *Econometrica 70*(2), 481–517.

Caucutt, E., L. Lochner, J. Mullins, and Y. Park (2026). Child Skill Production: Accounting for Parental and Market-Based Time and Goods Investments. *Journal of Political Economy 134*(1), 150–209.

Chetty, R., J. N. Friedman, and J. E. Rockoff (2014). Measuring the Impacts of Teachers II: Teacher Value-Added and Student Outcomes in Adulthood. *The American Economic Review 104*(9), 2633–2679.

Cunha, F. and J. Heckman (2007). The Technology of Skill Formation. *The American Economic Review 97*(2), 31–47.

Cunha, F., J. J. Heckman, L. Lochner, and D. V. Masterov (2006). Interpreting the Evidence on Life Cycle Skill Formation. In E. A. Hanushek and F. Welch (Eds.), *Handbook of the Economics of Education*, Volume 1, Chapter 12, pp. 697–812. North-Holland.

Cunha, F., J. J. Heckman, and S. M. Schennach (2010). Estimating the Technology of Cognitive and Noncognitive Skill Formation. *Econometrica 78*(3), 883–931.

Fuchs-Schündeln, N., D. Krueger, A. Kurmann, E. Lalé, A. Ludwig, and I. Popova (2023). The Fiscal and Welfare Effects of Policy Responses to the Covid-19 School Closures. *IMF Economic Review 71*(1), 35–98.

Fuchs-Schündeln, N., D. Krueger, A. Ludwig, and I. Popova (2022). The Long-Term Distributional and Welfare Effects of Covid-19 School Closures. *The Economic Journal 132*(645), 1647–1683.

Glomm, G. and B. Ravikumar (1992). Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality. *Journal of Political Economy 100*(4), 818–834.

Kotera, T. and A. Seshadri (2017). Educational Policy and Intergenerational Mobility. *Review of Economic Dynamics 25*, 187–207.

Krueger, A. B. (1999). Experimental Estimates of Education Production Functions. *The Quarterly Journal of Economics 114*(2), 497–532.

Lee, S. Y. and A. Seshadri (2019). On the Intergenerational Transmission of Economic Status. *Journal of Political Economy 127*(2), 855–921.

OECD (2026). *Education at a Glance 2026: OECD Indicators*. Paris: OECD Publishing.